SHIP ROLL BEHAVIOUR IN LARGE AMPLITUDE BEAM WAVES

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Overview

- Background
  - Ship Roll motion & Large waves
- Mathematical Model
  - Nonlinear ship rolling system
  - Fully nonlinear regular wave system
- Analysis Methods
  - Homotopy analysis method
  - Numerical simulation
  - Floquet theory
- Results and Discussion
- Conclusions

Background

- Ship roll motion

Cargo

Crew

Hazardous circumstances

Capsizing

Most significant among ship motions

- Harsh ocean environments

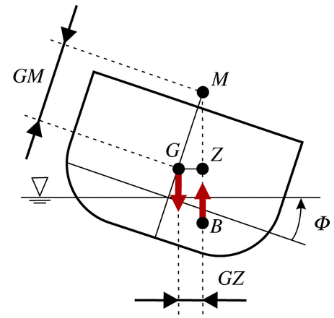
Highly nonlinear (such as large amplitude waves)

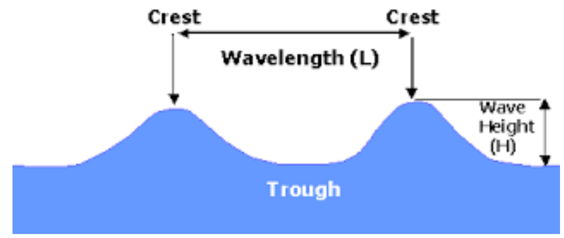
- Stability studies

Important to evaluate the stability of ship motions

Consider

Ship roll motion + Larger beam wave (regular)





Aim

1) modify the nonlinear rolling model

2) solve highly nonlinear system

3) stability analysis

Mathematical Model

Ship roll motions in beam waves

$$(I_{xx}+\delta I_{xx})\ddot{\phi}+D(\dot{\phi},\phi)+R(\phi)=M(t)$$

$$(I_{xx}+\delta I_{xx})=\omega_0^2\Delta GM$$

$$D(\dot{\phi},\phi)=D_1\dot{\phi}+D_3\dot{\phi}^3$$

$$R(\phi)=\Delta GM\phi+K_3\phi^3+K_5\phi^5$$

$$M(t)=-I_{xx}\ddot{\alpha}(t)$$

Linear exciting force

$$\eta(x,t)=A\cos(kx-\omega t)$$

$$\alpha(t)=kA\cos(\omega t)$$

$$M(t)=I_{xx}kA\omega^2\cos(\omega t)$$

Nonlinear exciting force

$$\eta(x,t)=\sum_{n=0}^{+\infty}a_n\cos n(kx-\omega t)$$

$$\alpha(t)=k\sum_{n=0}^{+\infty}a_n\cos n\omega t$$

$$M(t)=I_{xx}k\omega^2\sum_{n=1}^{+\infty}n^2a_n\cos(n\omega t)$$

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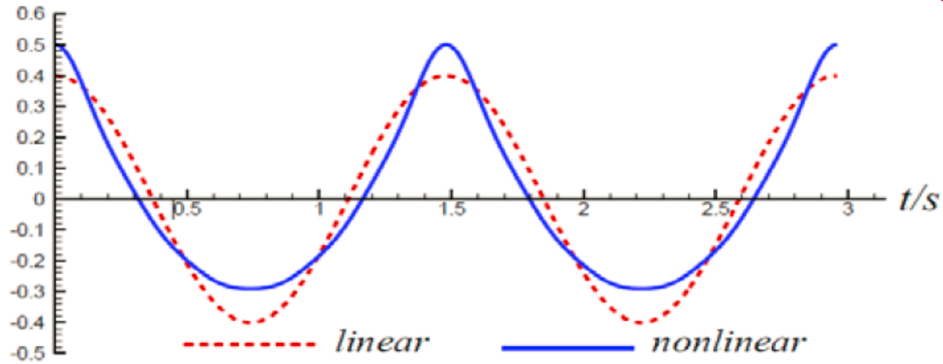
Nonlinear regular wave (wave theory)

$$\eta(x,t)=\sum_{n=0}^{+\infty} a_n \cos n(kx-\omega t)$$



1) fully nonlinear

2) including high order terms

Mathematical model for waves

$$\nabla^2\varphi=0, \quad -\infty<z<\eta,$$

$$\frac{\partial\eta}{\partial t}+\frac{\partial\varphi}{\partial x}\frac{\partial\eta}{\partial x}-\frac{\partial\varphi}{\partial z}=0, \quad \text{on } z=\eta,$$

$$\frac{\partial\varphi}{\partial t}+\frac{1}{2}\nabla\varphi\nabla\varphi+g\eta=0, \quad \text{on } z=\eta,$$

$$\frac{\partial\varphi}{\partial z}=0, \quad \text{as } z\to-\infty,$$

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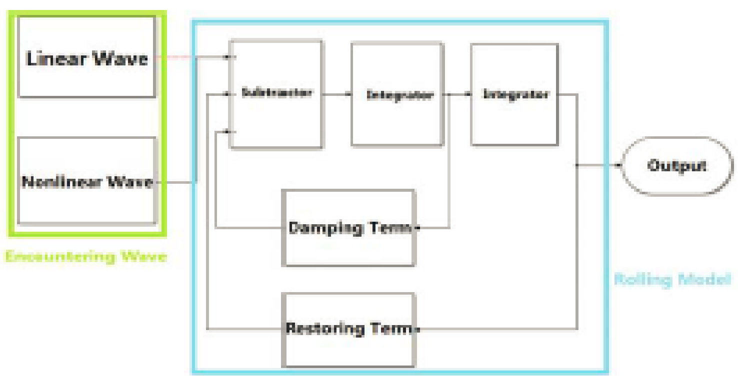
Analysis method

- Homotopy analysis method (HAM)

  effective for highly nonlinear ODE or PDE systems

  for solving wave system and rolling model

- Numerical simulation & Fitting



Simulink

+ Fourier series fitting

- Floquet theory

  for the stability analysis

HAM

$$N[u(t)] = 0$$

⇓

$$(1-p)L[\phi(t;p) - u_0(t)] = p\, c_0\, N[\phi(t;p)]$$

where

L → the linear operator, $L[0] = 0$

$u_0(t)$ → the initial estimate of $u(t)$

c_0 → the convergence-control parameter

⇓

$$L[u_m(t)] = c_0\, R_m(t, u_0, u_1, \cdots, u_{m-1})$$

⇓

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)$$

Advantages:

- ✓ **independent of small/large parameters;**
- ✓ **provides a convenient way to guarantee the convergence of solution series;**
- ✓ **provides great freedom to choose the equation type of linear sub-problems;**
- ✓ **valid for highly nonlinear problems**

Floquet theory

$$\bar{\phi}(t) = \phi(t) + \varepsilon(t)$$

$$\ddot{\varepsilon} + (d_1 + 3d_3\dot{\phi}^2)\dot{\varepsilon} + (\omega_0^2 + 3k_3\phi^2 + 5k_5\phi^4)\varepsilon = 0$$

$$\phi(t+T) = \phi(t) \qquad \Phi(t) = \begin{pmatrix} \varepsilon_1(t) & \dot{\varepsilon}_1(t) \\ \varepsilon_2(t) & \dot{\varepsilon}_2(t) \end{pmatrix}$$

$$\Phi(t+T) = C \cdot \Phi(t)$$

$$\Psi(t) = \begin{pmatrix} \xi_1(t) & \dot{\xi}_1(t) \\ \xi_2(t) & \dot{\xi}_2(t) \end{pmatrix}$$

$$\Phi(t) = P \cdot \Psi(t)$$

$$B = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$\Psi(t+T) = P^{-1}CP \cdot \Psi(t) = B \cdot \Psi(t)$$

$$\xi_1(t+T) = \lambda_1 \xi_1(t), \quad \xi_2(t+T) = \lambda_2 \xi_2(t)$$

$$\xi_1(t+nT) = \lambda_1^n \xi_1(t), \quad \xi_2(t+nT) = \lambda_2^n \xi_2(t)$$

Stability Criteria

$$\xi_i \to \begin{cases} 0, & \text{if} \quad |\lambda_i| < 1 \\ \infty, & \text{if} \quad |\lambda_i| > 1 \end{cases}$$

Results & discussion

Nondimensional roll motion equation:

$$\ddot{\phi} + d_1\dot{\phi} + d_3\dot{\phi}^3 + \omega_0^2\phi + k_3\phi^3 + k_5\phi^5 = \sum_{n=1}^{+\infty} F_n \cos(n\omega t)$$

Test vessel (Wright and Marshfield, 1980)

Coefficients

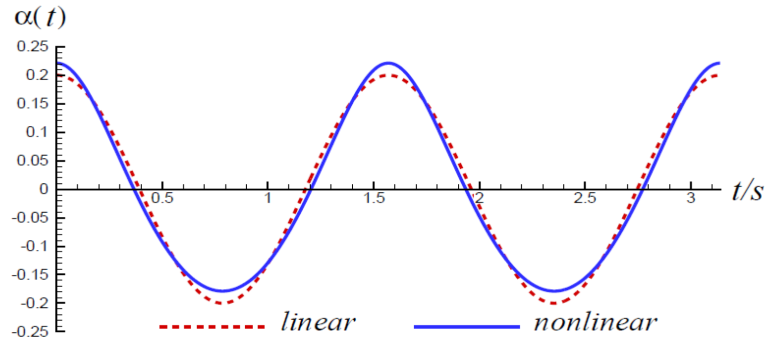
λ	0.8
$\omega_0(rad \cdot s^{-1})$	5.278
$d_1(rad \cdot s^{-1})$	0.171
$d_3(s)$	0.108
$k_3((rad \cdot s^{-1})^2)$	-39.056
$k_5((rad \cdot s^{-1})^2)$	7.549

[6]Right J.R.G., Marshfield W.B., 1980. Ship roll response and capsize behavior in beam seas, Trans. R. Inst. Nav. Archil., vol. 122, pp. 129-148.

Three kinds of beam waves ($L_w = 4$ m)

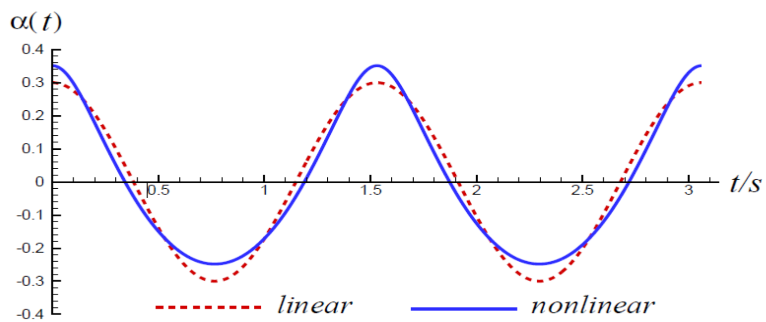
$$\alpha(t) = k\sum_{n=0}^{+\infty} a_n \cos n\omega t$$

Case 1



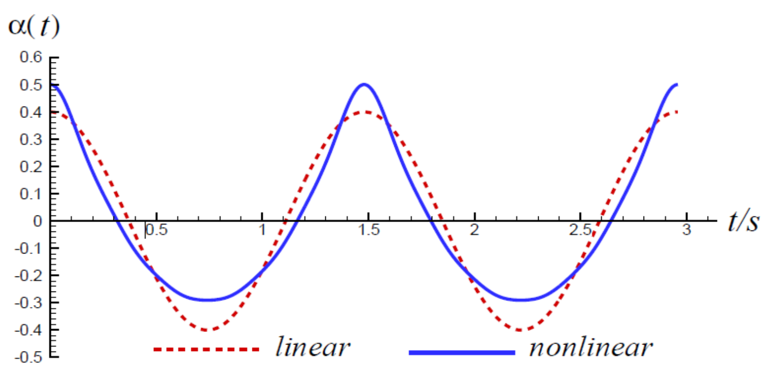
a_1	1.25164×10^{-1}	a_4	3.92717×10^{-4}
a_2	1.30581×10^{-2}	a_5	8.19823×10^{-5}
a_3	2.07357×10^{-3}	a_6	1.82049×10^{-5}
H/L_w	6.36620×10^{-2}	ω	4.00276

Case 2



a_1	1.82295×10^{-1}	a_4	2.39072×10^{-3}
a_2	3.01474×10^{-2}	a_5	8.15455×10^{-4}
a_3	7.74436×10^{-3}	a_6	2.95614×10^{-4}
H/L_w	9.54930×10^{-2}	ω	4.10404

Case 3



a_1	2.25563×10^{-1}	a_4	9.93992×10^{-3}
a_2	5.39358×10^{-2}	a_5	5.26656×10^{-3}
a_3	2.09279×10^{-2}	a_6	2.98442×10^{-3}
H/L_w	1.27324×10^{-1}	ω	4.24652

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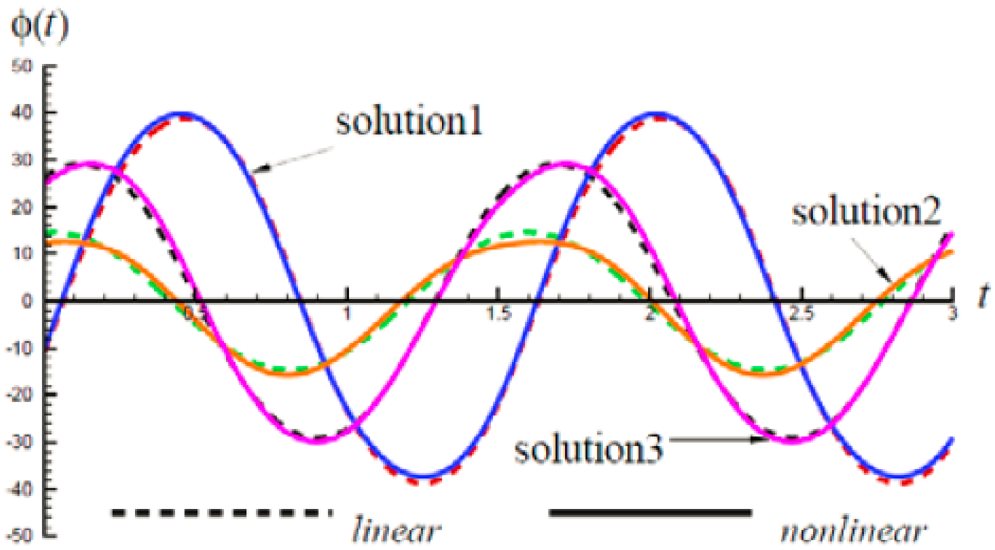
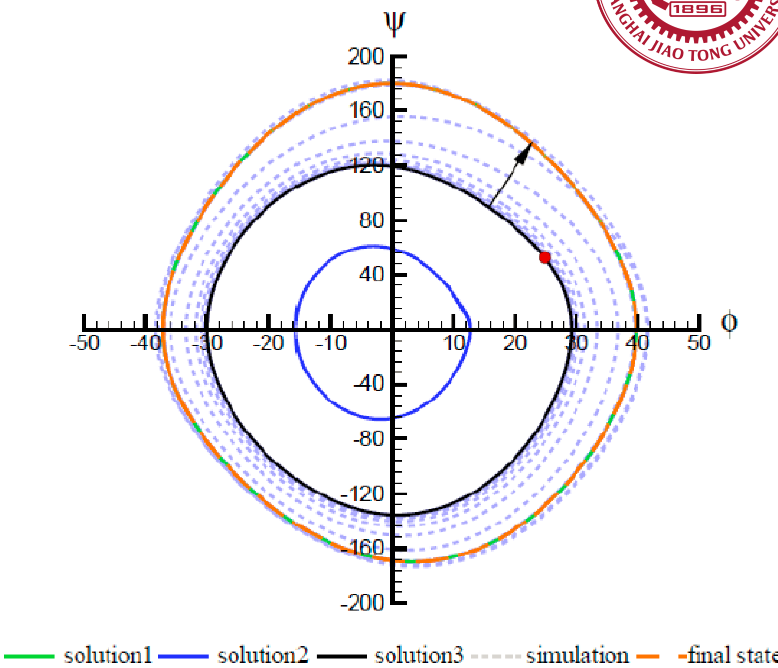
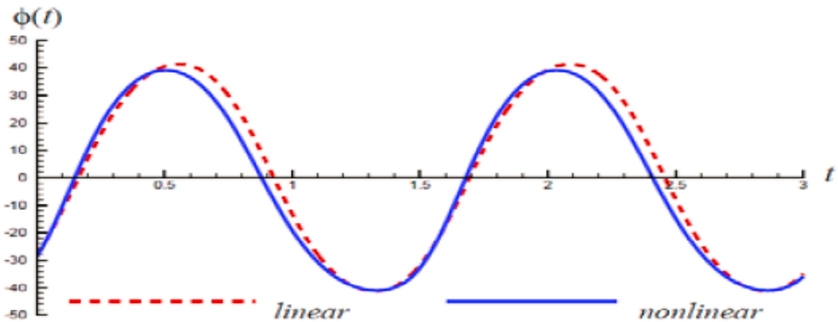
Case 1





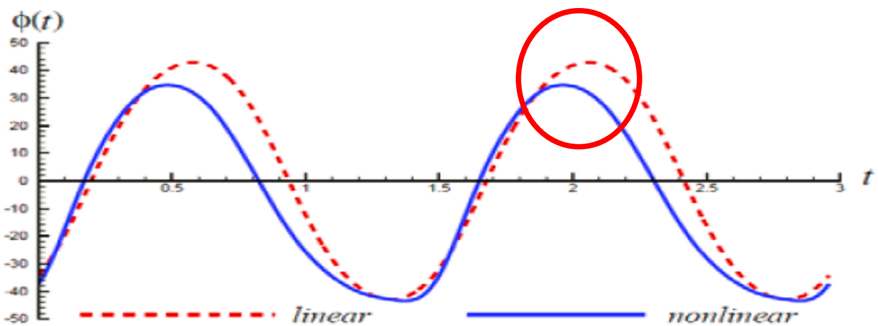
		ϕ_{max}	ϕ_{min}	$\|\lambda_1\|$	$\|\lambda_2\|$
solution1	linear	38.712	-38.712	0.325	0.325
	nonlinear	39.678	-37.295	0.326	0.326
solution2	linear	14.628	-14.628	0.765	0.765
	nonlinear	12.660	-15.686	0.765	0.765
solution3	linear	29.088	-29.088	1.868	0.137
	nonlinear	29.107	-29.940	1.865	0.133

Case 2

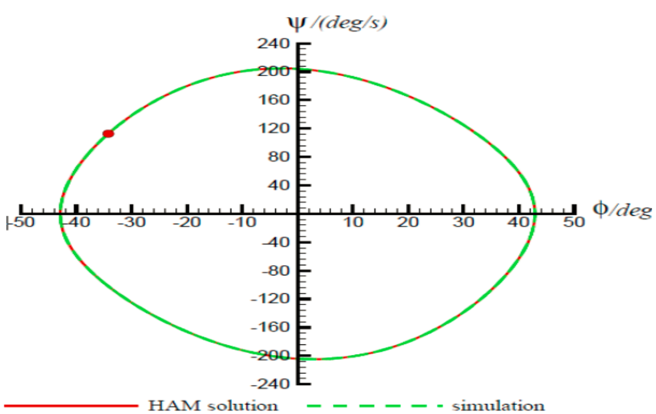


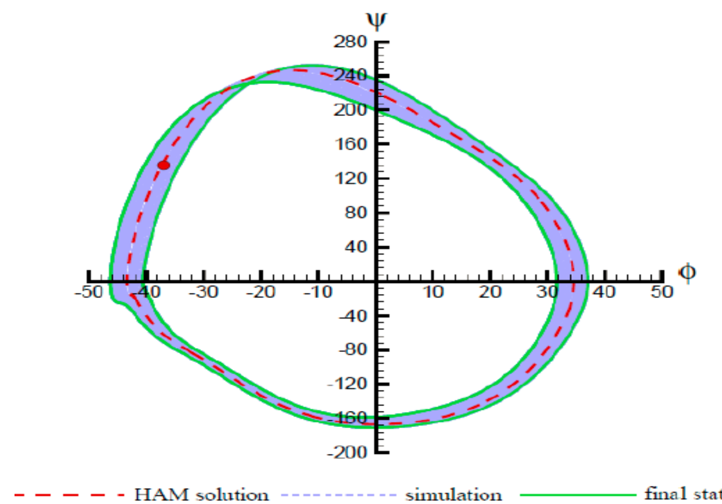
	ϕ_{max}	ϕ_{min}	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$
linear	41.2846	-41.2846	0.276	0.276
nonlinear	39.0820	-41.0962	0.662	0.122

Case 3



	ϕ_{max}	ϕ_{min}	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$
linear	42.8849	-42.8849	0.243	0.243
nonlinear	34.6369	-43.3705	1.072	0.071





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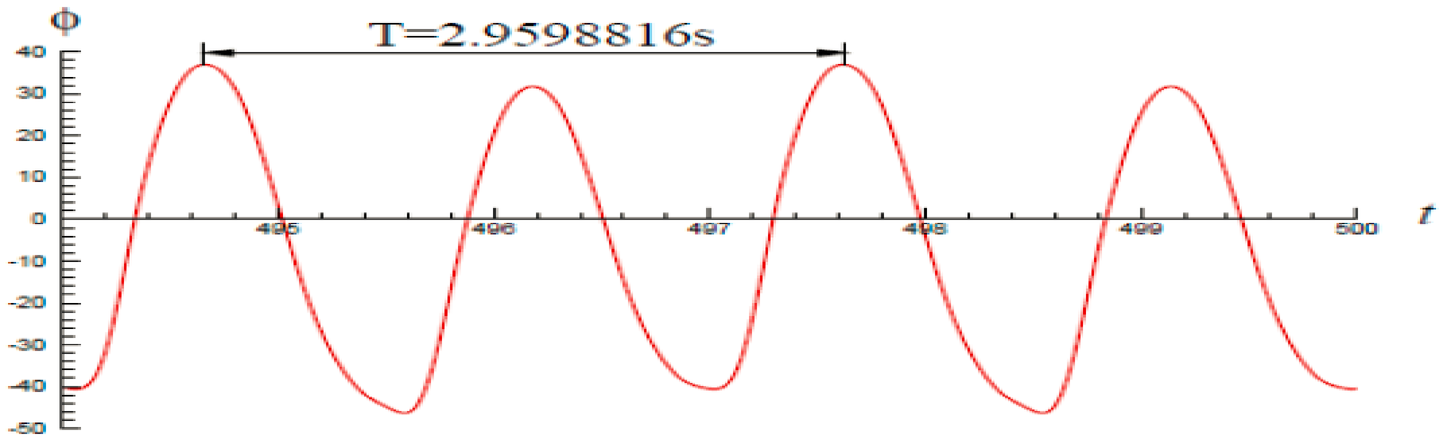
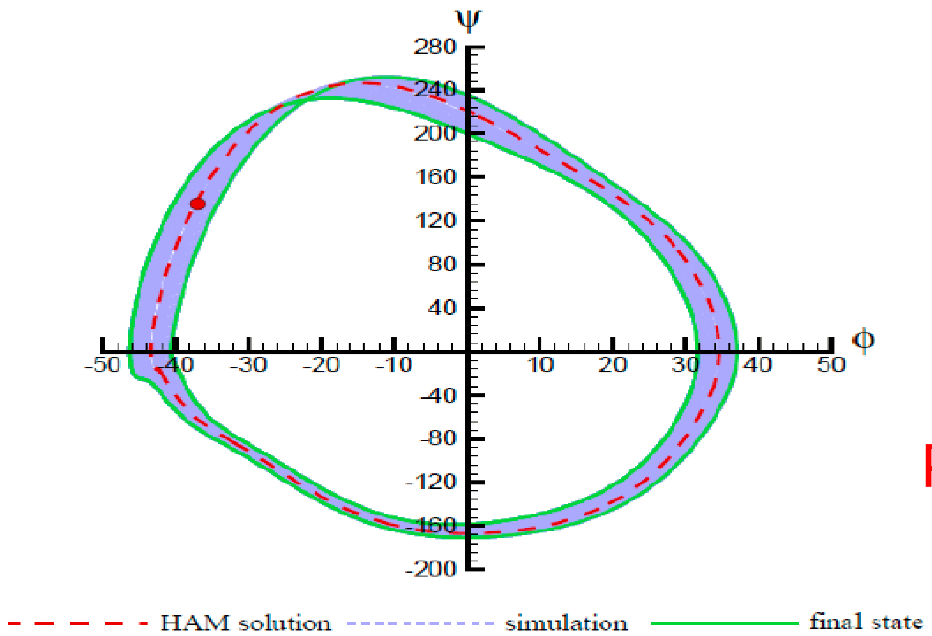
Case 3

Numerical simulation





$$T = 2.9598816s$$

wave period

$$T = 2\pi / \omega = 1.4796081s$$

Period doubling bifurcation

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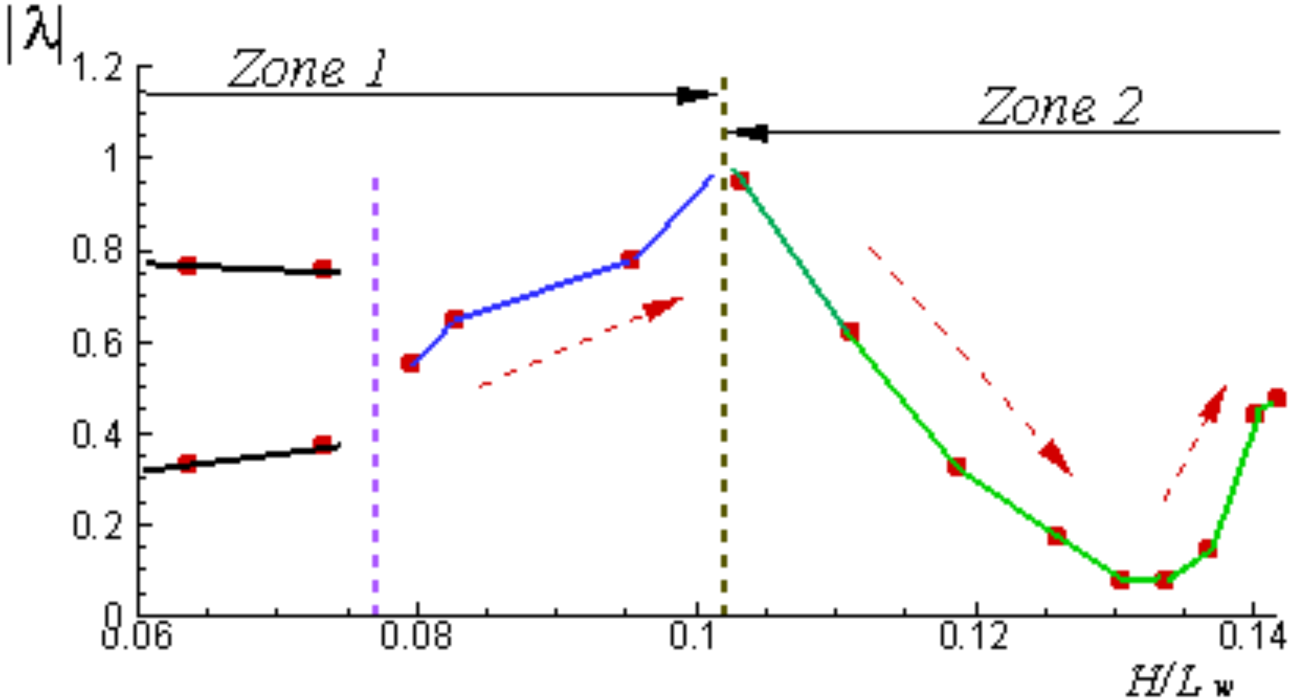
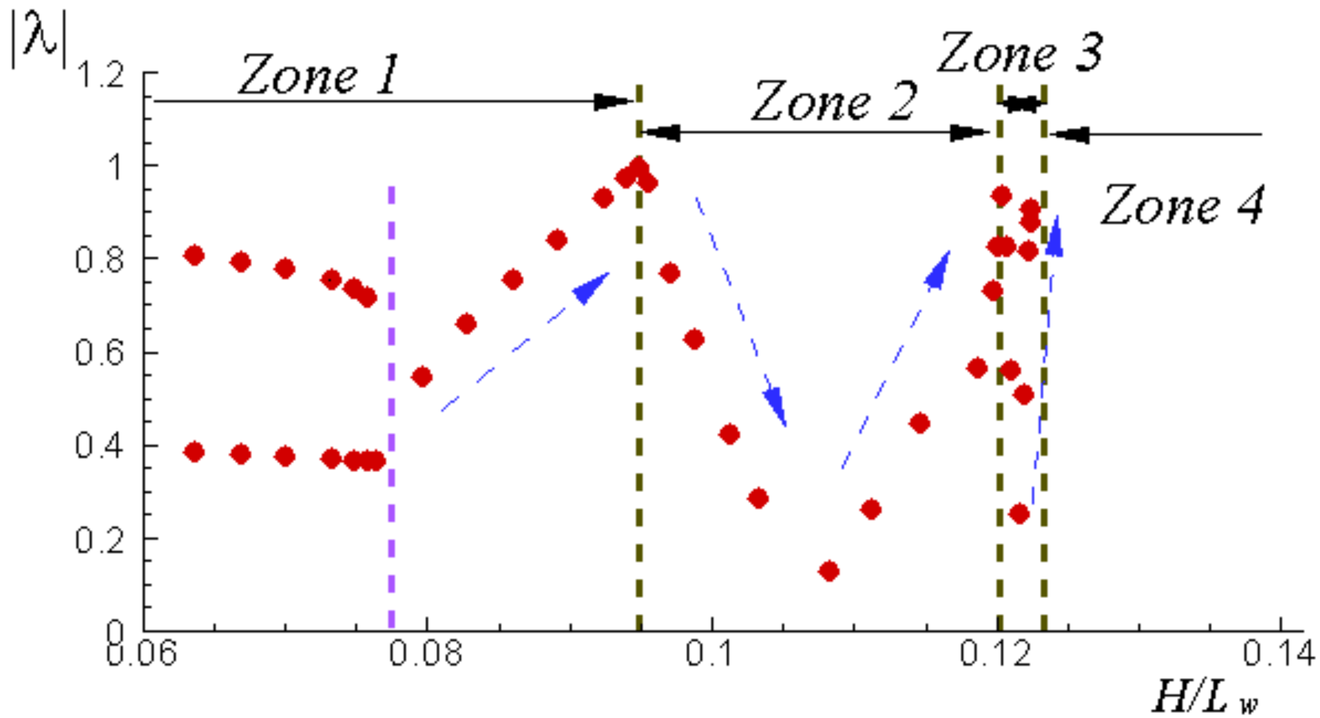
Keep the frequency of incident beam wave unchanged (ω =4.0)



Zone 1: $T_{roll} = T_w$

Zone 2: $T_{roll} = 2T_w$

Let $d_1^* = 0.8d_1$ and $d_3^* = 0.8d_3$



Zone 1: $T_{roll} = T_w$

Zone 2: $T_{roll} = 2T_w$

Zone 3: $T_{roll} = 4T_w$

Zone 4: Chaos

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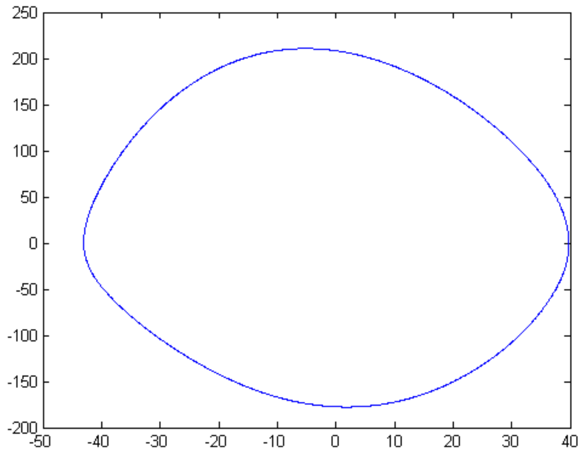
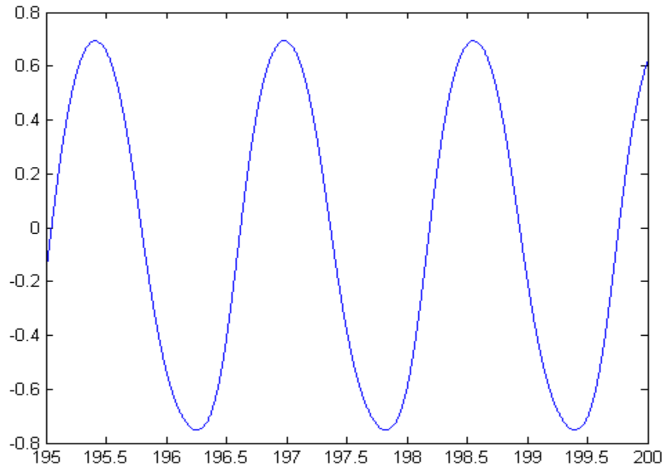
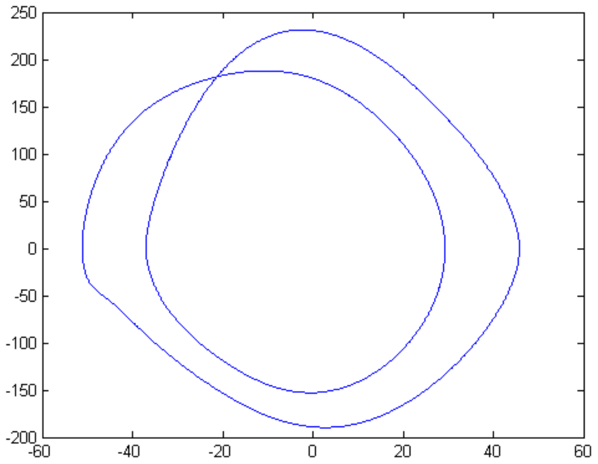
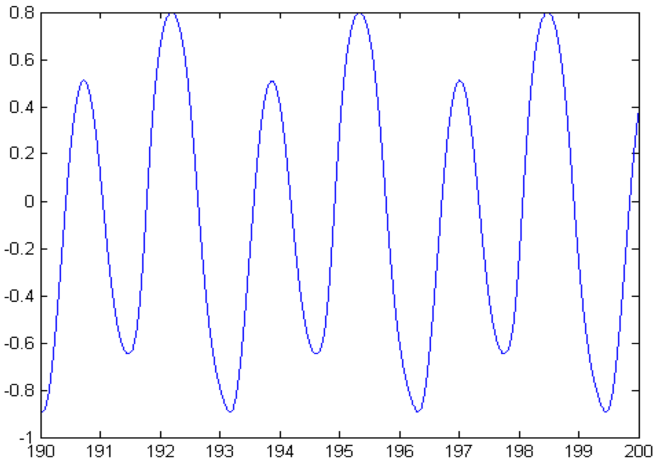
Zone 1: $T_{roll} = T_w$





Zone 2: $T_{roll} = 2T_w$





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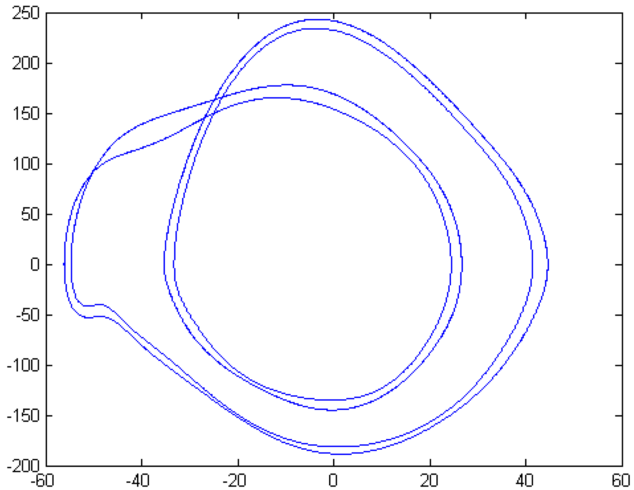
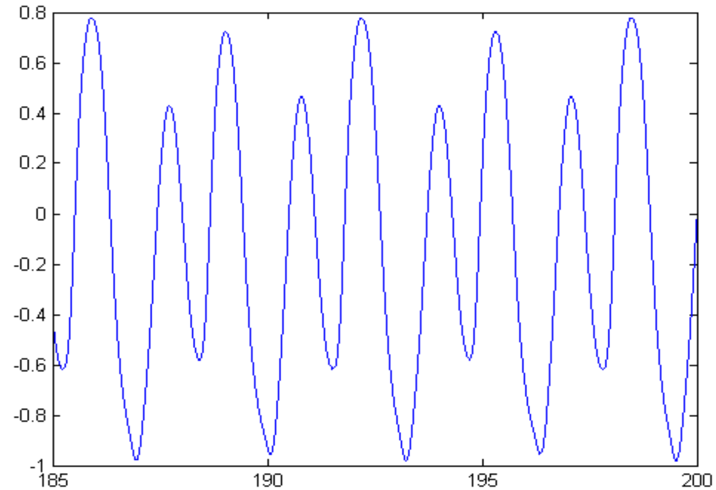
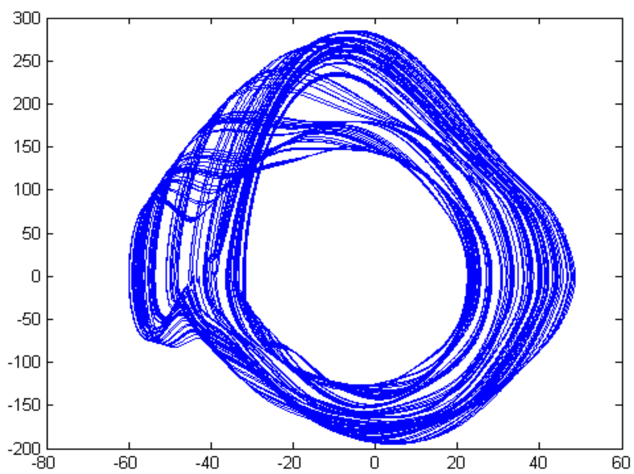
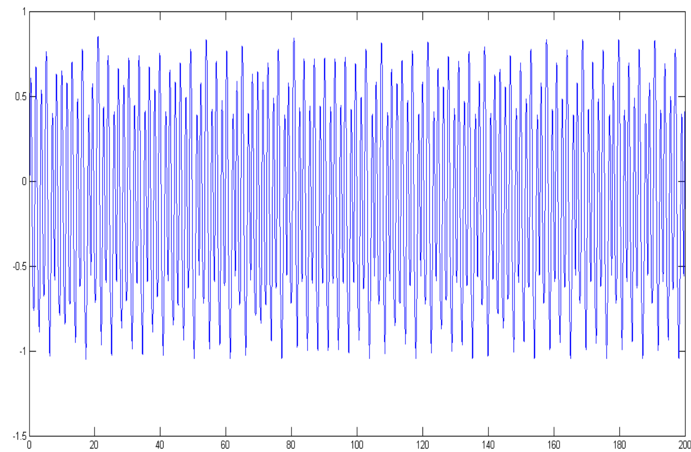
Zone 3: $T_{roll} = 4T_w$





Zone 4: Chaos





Conclusions

- When the slope of incident wave is small, there are two stable solutions of roll responses;
- As the wave slope increases, the difference of roll motions between linear and nonlinear wave exciting force gets more significant;
- When the wave slope is large enough, the roll amplitude doesn't increase all the time, but the roll motion will experience period doubling, quadrupling bifurcation, chaos, and finally the ship capsizes;
- The high order harmonic force terms cannot be neglected when considering ship roll motion in large beam waves.


Thanks for your
kind attention!